Tasks of “Mission impossible” and “Mission impeded” types

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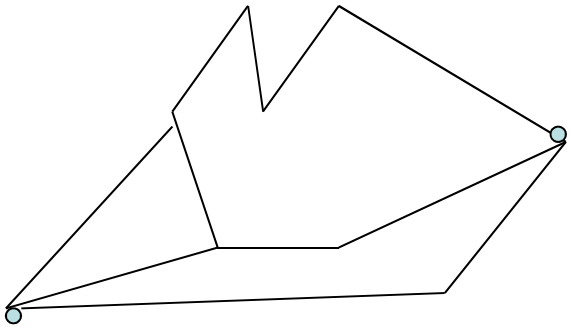
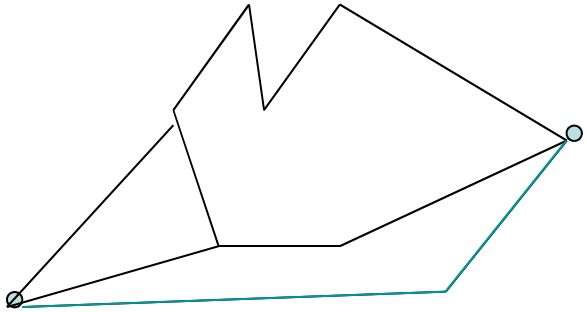
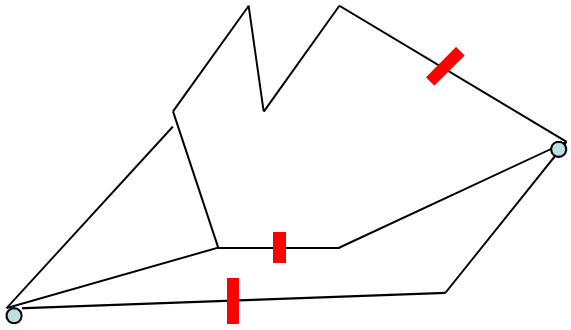
Environment



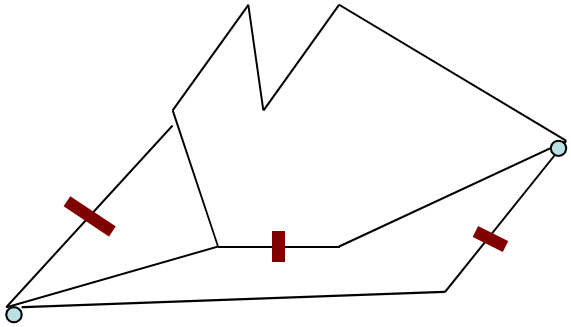
Mission



Mission impossible



Mission impeded



We propose a way to derive new tasks by reversing the goals of existing tasks as follows. Some operation is possible now and it can be performed in some number of steps.

Make this operation impossible (or to increase the number of steps to complete it). What is the minimum number of steps to achieve this goal?

Most of tasks given earlier ("*positive*"):

TA) to detect whether the object exists (or the operation is possible; the aim is attainable) (*alternative*);

TC) to find the number of cases (*combinatory*);

TO) to find the extreme value, the minimum number of steps (*optimization*);

TB) to build the object.

We propose the following pendants:
TA-A) to detect whether the possible operation can be made impossible (in exactly N steps)?
TA-L) to find the minimum number of steps to make the possible operation impossible; TC-L) . . .less than a given number;
TC-O) to make the number of cases the least possible by means of the given number of steps;
TO-L) to find the minimum number of steps to make the extreme value worse than a given number (boundary);
TO-O) to make the extreme value the worst possible by means of the given number of steps.

Task 1 (TA-L). (ACM ICPC KGZ quarterfinal, 2010). Given a word *W* of 4..100 letters. How many letters must be erased from *W* to make the word '*SU*' not obtainable from the rest of *W* by means of further erasing letters?
Call such relation "*S-embedding*" of '*SU*' into *W*.
Example. *BSSSSKKRRSSUU* → *2*.
Solution. *If (not(S in W)) or (not(U in W)) then Output 0 else {M:= min { number of S in W; number of U in W};*
for all clearances C in W { M1:= (number of S in W left to C) + (number of U in W right to C); M:=min{M,M1}}; Output M}.

N:=length(W). If *M1* is counted directly then the complexity is $O(N^2)$; if preceding values of numbers are used then the complexity is *O(N).*

Task 2 (classical, as basic). Given the set *S* of words and the word *W*. Can *W* be composed of a subset of *S* under the condition A) without overlapping or B) with possible overlapping?

Negative tasks in the environment of Task 2:

Task 3 (TA-A). Can *K* words be removed from *S* to make such composing impossible?

Task 4 (TA-L). How many, at least, words must be removed from *S* to make such composing impossible?

If all words are made of a same (one) letter then

Task 5 (classical, as basic). Given a set *S* of natural numbers and a number *N*. Can *N* be presented as the sum of a subset of *S*?

Negative tasks in the environment of Task 5.

Task 6 (TA-A). Can *K* numbers be removed from *S* to make such presentation impossible?

Task 7 (TA-L). How many, at least, numbers must be removed from *S* to make such presentation impossible?

Task 9 (TA-L) (classical, negative in our terminology). Given a connected graph and two of its vertices (which are not subject for removal). How many, at least, A) arcs or B) vertices are to be removed to disconnect these two?

Other types of tasks in this environment:

Task 10 (TO-L) . . . to make the distance between these vertices greater than a given number?

Task 11 (TO-O) . . . to make the distance between these vertices as large as possible?

Task 12 (TC-L) . . . to make the number of paths implementing the distance between these vertices less than a given number?

An example of “negative“ task given earlier.

Idea of task 13 ”Training” (IOI’2007). Mirko and Slavko are training for the tandem cycling marathon in Croatia. They need to choose a route to train on. ... Riding in the back seat is easier. Because of this, Mirko and Slavko change seats in every city. To ensure that they get the same amount of training, they must choose a route with an even number of roads.

Mirko and Slavko's competitors decided to block some of the unpaved roads, making it impossible to find a training route satisfying the above requirements. There is a cost (a positive integer) of blocking each unpaved the road. Find the smallest total cost needed to block some roads ...

Conclusion

Probably, among the vast scope of tasks given at numerous competitions in informatics there were ones which could be considered as „negative“ in our terminology.

We propose to develop such tasks systematically.

We hope that successful application of methods proposed above would yield new tasks with "short and elegant formulation" (Dagiene et al., 2007), and being interesting to solve. This would enlarge the scope of tasks involved into olympiads.

Thank you for attention!