THE VALLEY OF MEXICO (Expected solution)

Authors: Francisco Zaragoza (México), Feliu Zagols (México), Criel Merino (México)

OPTIMAL SOLUTION: Suppose that you want the trade route to start in city *x*. It can only progress to its neighbor city on the left or on the right, because any other connection would divide the lake into two regions with unreached cities on both sides, making the construction of a non-crossing trade route impossible. Similar reasoning can be applied to the following cities. Every new city must be adjacent to the already connected cities. In this way, it can only be chosen between the two cities that are adjacent to the already connected ones.

It can also be seen that once you have a set of cities connected with a route that ends in city *y* it really doesn't matter the order in which the cities were connected. For example, lets say that you have a route that connects cities 2,3,4 and 5 and ends in city 5. It doesn't matter if the route goes (2-3-4-5), (3-2-4-5) or (4-3-2-5). For any of the previous routes the two cities that can be chosen next are either 1 or 6. This situation is distinctive of dynamic programming.

For any pair of cities (*u,v*) we say that *left*(*u,v*) is true if it is possible to construct a route that connects every city to the left of a line drawn from *u* to *v*, including *u* and *v* that ends in *v*. Also we say that *right*(*u,v*) is true if it is possible to construct a route that connects every city to the right that ends in *v*. We initialize the recursion stating that for every *i*, *left*(*i,i*)=*right*(*i,i*)=*true*. The recursion formulas are:

- *left*(*u,v*)=(*left*(*u,v-1*) and *agreement*(*v-1,v*)) or (*right*(*v-1,u*) and *agreement*(*u,v*))
- *right*(*u,v*)=(*right*(*u,v+1*) and *agreement*(*v+1,v*)) or (*left*(*v+1,u*) and *agreement*(*u,v*))

If you find a pair of cities (*i,j*) for which (*rigth*(*i,j*) and *left*(*i+1,j*)) = *true*, you have a solution. The route can be reconstructed easily using the same recursion.

Test run #	Group/Value	*cities*	commercial agreements
1	1/0	7	9
2	2/30	8	14
3	2/30	8	14
4	2/30	20	190
5	2/30	15	22
6	2/30	10	13
7	2/30	8	8
8	3/10	20	100
9	3/10	20	120
10	4/5	250	21021
16	4/5	250	21031
17	5/5	350	40615
18	5/5	400	53125
11	6/5	500	84500
19	6/5	500	82174
20	7/5	650	145956
21	8/5	750	190034
22	9/5	850	238841
23	10/5	900	271903
24	11/5	950	310937
12	12/20	1000	400000
13	12/20	1000	331863
14	12/20	1000	342500
15	12/20	1000	341379